

Algorithm 3.2.1 Finding the Maximum of Three Numbers.

Input: Three numbers a , b , and c

Output: x , the largest of a , b , and c

procedure $max(a, b, c)$

$x := a$

if $b > x$ **then**

$x := b$

if $c > x$ **then**

$x := c$

return(x)

end max

Algorithm 3.2.2 Finding the Largest Element in a Finite Sequence.

Input: The sequence s_1, s_2, \dots, s_n and the length n of the sequence

Output: *large*, the largest element in this sequence

```
procedure find_large( $s, n$ )  
  large :=  $s_1$   
   $i := 2$   
  while  $i \leq n$  do  
    begin  
      if  $s_i > large$  then  
        large :=  $s_i$   
       $i := i + 1$   
    end  
  return(large)  
end find_large
```

Algorithm 3.2.3 Finding the Largest Element in a Finite Sequence.

Input: The sequence s_1, s_2, \dots, s_n and the length n of the sequence

Output: $large$, the largest element in this sequence

```
procedure find_large( $s, n$ )  
   $large := s_1$   
  for  $i := 2$  to  $n$  do  
    if  $s_i > large$  then  
       $large := s_i$   
  return( $large$ )  
end find_large
```

Algorithm 3.2.4 Testing Whether a Positive Integer is Prime.

Input: m , a positive integer

Output: **true**, if m is prime; **false**, if m is not prime

```
procedure is_prime( $m$ )  
  for  $i := 2$  to  $m - 1$  do  
    if  $m \bmod i = 0$  then  
      return(false)  
  return(true)  
end is_prime
```

Algorithm 3.2.5 Finding a Prime Larger Than a Given Integer.

Input: n , a positive integer

Output: m , the smallest prime greater than n

```
procedure large_prime( $n$ )  
   $m := n + 1$   
  while not is_prime( $m$ ) do  
     $m := m + 1$   
  return( $m$ )  
end large_prime
```

Algorithm 3.3.7 Euclidean Algorithm.

Input: a and b (nonnegative integers, not both zero)

Output: Greatest common divisor of a and b

```
procedure  $gcd(a, b)$   
  if  $a < b$  then  
     $swap(a, b)$   
  while  $b \neq 0$  do  
    begin  
       $r := a \bmod b$   
       $a := b$   
       $b := r$   
    end  
  return( $a$ )  
end  $gcd$ 
```

Algorithm 3.4.2 Computing n Factorial.

Input: n , an integer greater than or equal to 0

Output: $n!$

```
procedure factorial( $n$ )  
  if  $n = 0$  then  
    return(1)  
  return( $n * \textit{factorial}(n - 1)$ )  
end factorial
```

Algorithm 3.4.5 Recursively Computing the Greatest Common Divisor.

Input: a and b (nonnegative integers, not both zero)

Output: Greatest common divisor of a and b

```
procedure gcd_rekurs( $a, b$ )  
  if  $a < b$  then  
    swap( $a, b$ )  
  if  $b = 0$  then  
    return( $a$ )  
   $r := a \bmod b$   
  return(gcd_rekurs( $b, r$ ))  
end gcd_rekurs
```


Algorithm 3.4.7 Robot Walking.Input: n Output: $\text{walk}(n)$

```
procedure robot_walk( $n$ )  
  if  $n = 1$  or  $n = 2$  then  
    return( $n$ )  
  return(robot_walk( $n - 1$ ) + robot_walk( $n - 2$ ))  
end robot_walk
```

Algorithm 3.5.16 Searching an Unordered Sequence.

Input: s_1, s_2, \dots, s_n, n , and key (the value to search for)

Output: The location of key , or if key is not found, 0

```
procedure linear_search( $s, n, key$ )  
  for  $i := 1$  to  $n$  do  
    if  $key = s_i$  then  
      return( $i$ )  
  return(0)  
end linear_search
```

Algorithm 4.3.9 Generating Combinations.

Input: r, n

Output: All r -combinations of $\{1, 2, \dots, n\}$ in increasing lexicographic order.

```
procedure combination( $r, n$ )  
  for  $i := 1$  to  $r$  do  
     $s_i := i$   
  print  $s_1, \dots, s_r$   
  for  $i := 2$  to  $C(n, r)$  do  
    begin  
       $m := r$   
       $max\_val := n$   
      while  $s_m = max\_val$  do  
        begin  
           $m := m - 1$   
           $max\_val := max\_val - 1$   
        end  
       $s_m := s_m + 1$   
      for  $j := m + 1$  to  $r$  do  
         $s_j := s_{j-1} + 1$   
      print  $s_1, \dots, s_r$   
    end  
end combination
```

Algorithm 4.3.14 Generating Permutations.Input: n Output: All permutations of $\{1, 2, \dots, n\}$ in increasing lexicographic order.

```
procedure permutation( $n$ )  
  for  $i := 1$  to  $n$  do  
     $s_i := i$   
  print  $s_1, \dots, s_n$   
  for  $i := 2$  to  $n!$  do  
    begin  
       $m := n - 1$   
      while  $s_m > s_{m+1}$  do  
         $m := m - 1$   
       $k := n$   
      while  $s_m > s_k$  do  
         $k := k - 1$   
      swap( $s_m, s_k$ )  
       $p := m + 1$   
       $q := n$   
      while  $p < q$  do  
        begin  
          swap( $s_p, s_q$ )  
           $p := p + 1$   
           $q := q - 1$   
        end  
      print  $s_1, \dots, s_n$   
    end  
end permutation
```

Algorithm 5.3.1 Selection Sort.

Input: s_1, s_2, \dots, s_n and the length n of the sequence

Output: s_1, s_2, \dots, s_n , arranged in increasing order

procedure *selection_sort*(s, n)

if $n = 1$ **then**

return

$max_index := 1$

for $i := 2$ **to** n **do**

if $s_i > s_{max_index}$ **then**

$max_index := i$

$swap(s_n, s_{max_index})$

call *selection_sort*($s, n - 1$)

end *selection_sort*

Algorithm 5.3.2 Binary Search.

Input: A sequence s_i, s_{i+1}, \dots, s_j , $i \geq 1$, sorted in increasing order, a value key , i , and j

Output: The output is an index k for which $s_k = key$, or if key is not in the sequence, the output is the value 0.

```
procedure binary_search( $s, i, j, key$ )  
  if  $i > j$  then  
    return(0)  
   $k := \lfloor (i + j) / 2 \rfloor$   
  if  $key = s_k$  then  
    return( $k$ )  
  if  $key < s_k$  then  
     $j := k - 1$   
  else  
     $i := k + 1$   
  return(binary_search( $s, i, j, key$ ))  
end binary_search
```

Algorithm 5.3.5 Merging Two Sequences.

Input: Two increasing sequences: s_i, \dots, s_m and s_{m+1}, \dots, s_j , and indexes i, m , and j

Output: The sequence c_i, \dots, c_j consisting of the elements s_i, \dots, s_m and s_{m+1}, \dots, s_j combined into one increasing sequence

procedure *merge*(s, i, m, j, c)

$p := i$

$q := m + 1$

$r := i$

while $p \leq m$ **and** $q \leq j$ **do**

begin

if $s_p < s_q$ **then**

begin

$c_r := s_p$

$p := p + 1$

end

else

begin

$c_r := s_q$

$q := q + 1$

end

$r := r + 1$

end

while $p \leq m$ **do**

begin

$c_r := s_p$

$p := p + 1$

$r := r + 1$

end

while $q \leq j$ **do**

begin

$c_r := s_q$

$q := q + 1$

$r := r + 1$

end
end merge

Algorithm 5.3.8 Merge Sort.

Input: s_i, \dots, s_j , i , and j

Output: s_i, \dots, s_j arranged in increasing order

```
procedure merge_sort( $s, i, j$ )  
  if  $i = j$  then  
    return  
   $m := \lfloor (i + j)/2 \rfloor$   
  call merge_sort( $s, i, m$ )  
  call merge_sort( $s, m + 1, j$ )  
  call merge( $s, i, m, j, c$ )  
  for  $k := i$  to  $j$  do  
     $s_k := c_k$   
end merge_sort
```

Algorithm 6.4.1 Dijkstra's Shortest-Path Algorithm.

Input: A connected, weighted graph in which all weights are positive.
Vertices a and z .

Output: $L(z)$, the length of a shortest path from a to z .

```
procedure dijkstra( $w, a, z, L$ )  
   $L(a) := 0$   
  for all vertices  $x \neq a$  do  
     $L(x) := \infty$   
   $T :=$  set of all vertices  
  while  $z \in T$  do  
    begin  
    choose  $v \in T$  with minimum  $L(v)$   
     $T := T - \{v\}$   
    for each  $x \in T$  adjacent to  $v$  do  
       $L(x) := \min\{L(x), L(v) + w(v, x)\}$   
    end  
end dijkstra
```

Algorithm 7.1.9 Constructing an Optimal Huffman Code.

Input: A sequence of n frequencies, $n \geq 2$

Output: A rooted tree that defines an optimal Huffman code

procedure *huffman*(f, n)

if $n = 2$ **then**

begin

 let f_1 and f_2 denote the frequencies

 let T be as in Figure 7.1.11

return(T)

end

 let f_i and f_j denote the smallest frequencies

 replace f_i and f_j in the list f by $f_i + f_j$

$T' := \text{huffman}(f, n - 1)$

 replace a vertex in T' labeled $f_i + f_j$ by the tree shown in Figure 7.1.12

 to obtain the tree T

return(T)

end *huffman*

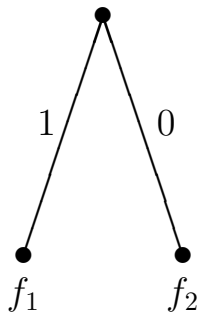


Figure 7.1.11

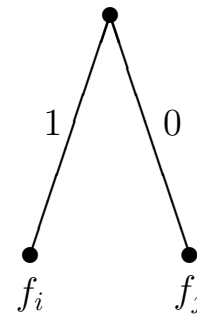


Figure 7.1.12

Algorithm 7.3.6 Breadth-First Search for a Spanning Tree.

Input: A connected graph G with vertices ordered v_1, v_2, \dots, v_n

Output: A spanning tree T

procedure $dfs(V, E)$

$S := (v_1)$

$V' := \{v_1\}$

$E' := \emptyset$

while true do

begin

for each $x \in S$, in order, **do**

for each $y \in V - V'$, in order, **do**

if (x, y) is an edge **then**

add edge (x, y) to E' and y to V'

if no edges were added **then**

return (T)

$S :=$ children of S ordered consistently with the original vertex ordering

end

end dfs

Algorithm 7.3.7 Depth-First Search for a Spanning Tree.

Input: A connected graph G with vertices ordered v_1, v_2, \dots, v_n

Output: A spanning tree T

procedure $dfs(V, E)$

$V' := \{v_1\}$

$E' := \emptyset$

$w := v_1$

while true do

begin

while there is an edge (w, v) that when added to T does not create a cycle in T **do**

begin

 choose the edge (w, v_k) with minimum k that when added to T does not create a cycle in T

 add (w, v_k) to E'

 add v_k to V'

$w := v_k$

end

if $w = v_1$ **then**

return(T)

$w :=$ parent of w in T

end

end dfs

Algorithm 7.3.10 Solving the Four-Queens Problem Using Backtracking.

Input: An array row of size 4

Output: **true**, if there is a solution

false, if there is no solution

[If there is a solution, the k th queen is in column k , row $row(k)$.]

```
procedure four_queens(row)
   $k := 1$ 
   $row(1) := 0$ 
  while  $k > 0$  do
    begin
       $row(k) := row(k) + 1$ 
      while  $row(k) \leq 4$  and column  $k$ ,  $row(k)$  conflicts do
         $row(k) := row(k) + 1$ 
      if  $row(k) \leq 4$  then
        if  $k = 4$  then
          return(true)
        else
          begin
             $k := k + 1$ 
             $row(k) := 0$ 
          end
        else
           $k := k - 1$ 
        end
      return(false)
    end four_queens
```

Algorithm 7.4.3 Prim's Algorithm.

Input: A connected, weighted graph with vertices $1, \dots, n$ and start vertex s . If (i, j) is an edge, $w(i, j)$ is equal to the weight of (i, j) ; if (i, j) is not an edge, $w(i, j)$ is equal to ∞ (a value greater than any actual weight).

Output: The set of edges E in a minimal spanning tree

```

procedure prim( $w, n, s$ )
  for  $i := 1$  to  $n$  do
     $v(i) := 0$ 
   $v(s) := 1$ 
   $E := \emptyset$ 
  for  $i := 1$  to  $n - 1$  do
    begin
       $min := \infty$ 
      for  $j := 1$  to  $n$  do
        if  $v(j) = 1$  then
          for  $k = 1$  to  $n$  do
            if  $v(k) = 0$  and  $w(j, k) < min$  then
              begin
                 $add\_vertex := k$ 
                 $e := (j, k)$ 
                 $min := w(j, k)$ 
              end
             $v(add\_vertex) := 1$ 
             $E := E \cup \{e\}$ 
          end
        return( $E$ )
      end
    end prim

```

Algorithm 7.5.10 Constructing a Binary Search Tree.

Input: A sequence w_1, \dots, w_n of distinct words and the length n of the sequence

Output: A binary search tree T

```
procedure make_bin_search_tree( $w, n$ )  
  let  $T$  be the tree with one vertex, root  
  store  $w_1$  in root  
  for  $i := 2$  to  $n$  do  
    begin  
       $v := \textit{root}$   
      search := true  
      while search do  
        begin  
           $s := \textit{word in } v$   
          if  $w_i < s$  then  
            if  $v$  has no left child then  
              begin  
                add a left child  $l$  to  $v$   
                store  $w_i$  in  $l$   
                search := false  
              end  
            else  
               $v := \textit{left child of } v$   
            end  
          else  
            if  $v$  has no right child then  
              begin  
                add a right child  $r$  to  $v$   
                store  $w_i$  in  $r$   
                search := false  
              end  
            else  
               $v := \textit{right child of } v$   
            end  
          end  
        end  
      end  
    end  
  end
```



```
    return( $T$ )  
end make_bin_search_tree
```

Algorithm 7.6.1 Preorder Traversal.

Input: PT , the root of a binary tree

Output: Dependent on how “process” is interpreted

```
procedure preorder( $PT$ )  
  if  $PT$  is empty then  
    return  
  process  $PT$   
   $l :=$  left child of  $PT$   
  preorder( $l$ )  
   $r :=$  right child of  $PT$   
  preorder( $r$ )  
end preorder
```

Algorithm 7.6.3 Inorder Traversal.

Input: PT , the root of a binary tree

Output: Dependent on how “process” is interpreted

```
procedure inorder( $PT$ )  
  if  $PT$  is empty then  
    return  
   $l :=$  left child of  $PT$   
  inorder( $l$ )  
  process  $PT$   
   $r :=$  right child of  $PT$   
  inorder( $r$ )  
end inorder
```

Algorithm 7.6.5 Postorder Traversal.

Input: PT , the root of a binary tree

Output: Dependent on how “process” is interpreted

```
procedure postorder( $PT$ )  
  if  $PT$  is empty then  
    return  
   $l :=$  left child of  $PT$   
  postorder( $l$ )  
   $r :=$  right child of  $PT$   
  postorder( $r$ )  
  process  $PT$   
end postorder
```

Algorithm 7.8.13 Testing Whether Two Binary Trees Are Isomorphic.

Input: The roots r_1 and r_2 of two binary trees. (If the first tree is empty, r_1 has the special value *null*. If the second tree is empty, r_2 has the special value *null*.)

Output: **true**, if the trees are isomorphic
false, if the trees are not isomorphic

```
procedure bin_tree_isom( $r_1, r_2$ )  
  if  $r_1 = \textit{null}$  and  $r_2 = \textit{null}$  then  
    return(true)  
  if  $r_1 = \textit{null}$  or  $r_2 = \textit{null}$  then  
    return(false)  
   $lc\_r_1 :=$  left child of  $r_1$   
   $lc\_r_2 :=$  left child of  $r_2$   
   $rc\_r_1 :=$  right child of  $r_1$   
   $rc\_r_2 :=$  right child of  $r_2$   
  return(bin_tree_isom( $lc\_r_1, lc\_r_2$ ) and bin_tree_isom( $rc\_r_1, rc\_r_2$ ))  
end bin_tree_isom
```

Algorithm 8.2.4 Finding a Maximal Flow in a Network.

Input: A network with source a , sink z , capacity C , vertices $a = v_0, \dots, v_n = z$, and n

Output: A maximal flow F

```

procedure max_flow( $a, z, C, v, n$ )
  for each edge  $(i, j)$  do
     $F_{ij} := 0$ 
  while true do
    begin
      for  $i := 0$  to  $n$  do
        begin
           $predecessor(v_i) := null$ 
           $val(v_i) := null$ 
        end
       $predecessor(a) := -$ 
       $val(a) := \infty$ 
       $U := \{a\}$ 
      while  $val(z) = null$  do
        begin
          if  $U = \emptyset$  then
            return( $F$ )
          choose  $v$  in  $U$ 
           $U := U - \{v\}$ 
           $\Delta := val(v)$ 
          for each edge  $(v, w)$  with  $val(w) = null$  do
            if  $F_{vw} < C_{vw}$  then
              begin
                 $predecessor(w) := v$ 
                 $val(w) := \min\{\Delta, C_{vw} - F_{vw}\}$ 
                 $U := U \cup \{w\}$ 
              end
          for each edge  $(w, v)$  with  $val(w) = null$  do
            if  $F_{wv} > 0$  then
              begin

```

```

    predecessor(w) := v
    val(w) := min{Δ, Fwv}
    U := U ∪ {w}
  end
end
w0 := z
k := 0
while wk ≠ a do
  begin
    wk+1 := predecessor(wk)
    k := k + 1
  end
P := (wk, wk-1, ..., w1, w0)
Δ := val(z)
for i := 1 to k do
  begin
    e := (wi, wi-1)
    if e is properly oriented in P then
      Fe := Fe + Δ
    else
      Fe := Fe - Δ
    end
  end
end
end max_flow

```

Algorithm 11.1.2 Finding the Distance Between a Closest Pair of Points.

Input: p_1, \dots, p_n ($n \geq 2$ points in the plane)

Output: δ , the distance between a closest pair of points

```

procedure closest_pair( $p, n$ )
  sort  $p_1, \dots, p_n$  by  $x$ -coordinate
  return(rec_cl_pair( $p, 1, n$ ))
end closest_pair

procedure rec_cl_pair( $p, i, j$ )
  if  $j - i < 3$  then
    begin
      sort  $p_i, \dots, p_j$  by  $y$ -coordinate
      directly find the distance  $\delta$  between a closest pair
      return( $\delta$ )
    end
     $k := \lfloor (i + j) / 2 \rfloor$ 
     $l := p_k.x$ 
     $\delta_L := \text{rec\_cl\_pair}(p, i, k)$ 
     $\delta_R := \text{rec\_cl\_pair}(p, k + 1, j)$ 
     $\delta := \min\{\delta_L, \delta_R\}$ 
    merge  $p_i, \dots, p_k$  and  $p_{k+1}, \dots, p_j$  by  $y$ -coordinate
     $t := 0$ 
    for  $k := i$  to  $j$  do
      if  $p_k.x > l - \delta$  and  $p_k.x < l + \delta$  then
        begin
           $t := t + 1$ 
           $v_t := p_k$ 
        end
    for  $k := 1$  to  $t - 1$  do
      for  $s := k + 1$  to  $\min\{t, k + 7\}$  do
         $\delta := \min\{\delta, \text{dist}(v_k, v_s)\}$ 
      return( $\delta$ )
    end rec_cl_pair

```


Algorithm 11.3.6 Graham's Algorithm to Compute the Convex Hull.

Input: p_1, \dots, p_n and n

Output: p_1, \dots, p_k (the convex hull of p_1, \dots, p_n) and k

```

procedure graham_scan( $p, n, k$ )
  if  $n = 1$  then
    begin
       $k := 1$ 
      return
    end
   $min := 1$ 
  for  $i := 2$  to  $n$  do
    if  $p_i.y < p_{min}.y$  then
       $min := i$ 
  for  $i := 1$  to  $n$  do
    if  $p_i.y = p_{min}.y$  and  $p_i.x < p_{min}.x$  then
       $min := i$ 
  swap( $p_1, p_{min}$ )
  sort  $p_2, \dots, p_n$ 
   $p_0 := p_n$ 
   $k := 2$ 
  for  $i := 3$  to  $n$  do
    begin
      while  $p_{k-1}, p_k, p_i$  do not make a left turn do
         $k := k - 1$ 
       $k := k + 1$ 
      swap( $p_i, p_k$ )
    end
  end graham_scan

```